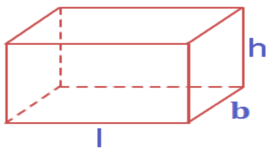
GRADE – 9

LESSON : 13 SURFACE AREAS AND VOLUMES

SURFACE AREA OF A CUBOID AND A CUBE

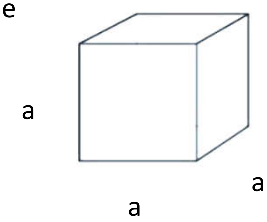
SOLIDS

Cuboid : Cuboid Outer Surface of a cuboid is made up of six rectangles called the faces of the cuboid.



Diagonal of cuboid $=\sqrt{l^2 + b^2 + h^2}$

Cube : Cuboid whose length, breadth and height are all equal is called cube



Diagonal of cube $= a\sqrt{3}$

Total surface area of a cuboid = 2 (lb + bh +

Lateral surface area of a cuboid = 2 (l + b) x h

Area of four walls of the room = 2 (l + b) x h

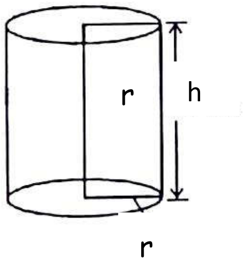
Total surface area of a cube = $6a^2$

Lateral surface area of a cube = $4a^2$

SURFACE AREA OF RIGHT CIRCULAR CYLINDER

RIGHT CIRCULAR CYLINDER

Where 'r' is the radius of the base of the cylinder and 'h' is height of the cylinder



Total surface area of a cylinder = $2\pi r$ (r + h)

Area of base of cylinder (bottom or top) circular shape = πr^2

Curved or lateral surface area of a cylinder = $2\pi r$h

Area of base ring = $\pi\,(R^2 - r^2)$
where R = External radius
r = Internal radius

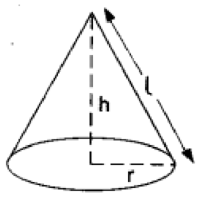
SURFACE AREA OF RIGHT CIRCULAR CONE

RIGHT CIRCULAR CONE

r = radius cone

h = height cone

l = slant height of cone = $\sqrt{r^2 + h^2}$



Curved surface area of a cone = $\pi r l$

Area of circular base of a cone = πr^2

Total surface area of a cone = $\pi r(r + l)$

Area of base ring = $\pi(R^2 - r^2)$

R = External radius; r = Internal radius

SURFACE AREA OF A SPHERE AND A HEMISPHERE

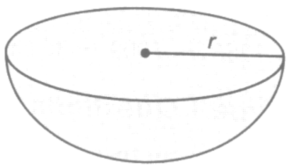
Sphere: Let 'r' be the radius of the sphere .

Surface area of a sphere = 4 πr^2

Hemisphere : Let 'r' be the radius of the hemisphere

Curved surface area of hemisphere = 2 πr^2

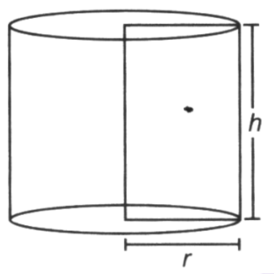
Total surface area of hemisphere = 2 πr^2 + πr^2 = 3 πr^2



VOLUME OF A CYLINDER AND A CONE

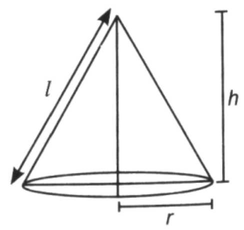
Volume of a cylinder = πr^2**h**

Where r is the base radius and h is the height of the cylinder



Volume of a Cone = $\frac{1}{3}\pi r^2$ **h**

Where is the base radius, h is the height of the cone and l is slant height of the cone



Grade IX

Lesson : 13 Surface Areas and Volumes

Objective Type Questions

13.1

I. Multiple choice questions

1. How do we judge that the given object is a solid object?

a) by touching it b) by finding its dimensions

c) distance between any two points of the object remains same

d) by keeping it on the table and noting down whether the whole object touches the table or not

Sol. d

2. Lateral surface area of a cuboid with dimensions l, b, h is.

a) 2(lb + bh + lh) – 2lb b) 2(lb + bh + lh)

c) 2 (l + b) h^2 d) $l$bh

Sol. a

3. Number of surfaces of the same area in a cuboid are.

a) 6 b) 4 c) 2 d) 3

Sol. c

4. Number of surfaces of the same area in a cube are.

a) 6 b) 4 c) 2 d) 3

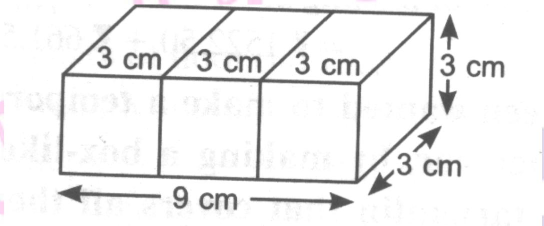
Sol. a

5. Three cubes are joined end to end forming a cuboid. If side of a cube is 3 cm, then dimensions of the cuboid are :

a) l = 2 , b = 2, h = 2 b) l = 4 , b = 4, h = 2

c) l = 4 , b = 2, h = 4 d) l = 9 , b = 3, h = 3

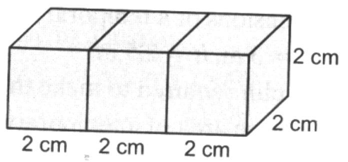
Sol. d



$l = 9$ cm; $b = 3$ cm: $h = 3$ cm

6. Three cubes are joined end to end forming a cuboid. If side of a cube is 2 cm, find the dimensions of cuboid thus obtained.

Given side of cube is 2 cm. Three cubes are joined end to end.



Dimensions of cuboid thus formed is (2 + 2 + 2) cm, 2cm, 2cm i.e., 6cm, 2cm, 2cm

7. Find the lateral surface area of cube, if its diagonal is $\sqrt{6}$ cm.

Sol. Let edge of cube be l

∴ Diagonal of a cube = $l\sqrt{3}$

⟹ $l\sqrt{3}$ = $\sqrt{6}$ ⟹ $\frac{\sqrt{6}}{\sqrt{3}}$ = 2 cm

⟹ Lateral surface area of a cube

= $4a^2$ = 4 x $\left(\sqrt{2}\right)^2$ = 8 cm^2

8. The dimensions of the cuboid are 5 cm x 4 cm x 2 cm. Find its diagonal.

Here, l = 5 cm, b = 4 cm h = 2 cm

∴ Diagonal of a cuboid

= $\sqrt{l^2 + b^2 + h^2}$ = $\sqrt{5^2 + 4^2 + 2^2}$

= $\sqrt{25 + 16 + 4}$ = $\sqrt{45}$ cm = $3\sqrt{5}$ cm

II. Multiple choice questions

1. Charvi has a rectangular sheet of paper and wants to fold it in such a manner that the cylinder thus formed should have maximum surface area. Then she should fold it,

   a) along length b) along breadth

   c) along any of the sides length or breadth

   Sol. c

2. A cylinder of base radius 'r' and height 'h' is dipped vertically to half the height, in a bucket full of purple paint, Then the area of the surface which gets painted is

   a) $\frac{1}{2}\pi r^2 h$ b) $\frac{1}{3}\pi r^2 h$ c) πrh d) πr (h + r)

   Sol. d

3. In a split A.C., the inner cooler is formed by a cooling pipe to the outer instrument. The length of the pipe is 3 m and diameter is $\frac{1}{2}$ cm. When A.C is switched on, then the surface of the pipe cooled in the process has area.

   a) $3\pi\ m^2$ b) 0.015 $\pi\ m^2$ c) 150 $\pi\ cm^2$ d) $300\ \pi\ m^2$

   Sol. b

III. Multiple choice questions

1. Slant height of a cone is 34 cm and base diameter is 32 cm then height of the cone is

   a) 33 cm b) 25 cm c) 30 cm d) 27 cm

   Sol. c

2. For Diwali celebrations, a packet of "Conical Diwas" is bought. In that, there are 15 conical diwas, with red glittering polish surface. Jaya took a scale and noted the following informations:

   Radius of base is 7cm and slant height is 10 cm. Then calculate how much total area is polished.

   a) 5160 cm^2 b) 5601 cm^2 c) 6501 cm^2 d) 5610 cm^2

   Sol. d

3. Find the total surface area of a cone whose radius is $\frac{r}{2}$ units and slant height is $2l$ units

Sol. Here Radius (R) = $\frac{r}{2}$ units, slant height (L) = 2 l units

Total surface area of cone = $\pi R\ (R + L)$

= $\pi \left(\frac{r}{2}\right)\left(\frac{r}{2} + 2l\right)$ = $\pi \left(\frac{r}{2}\right)\left(\frac{r+4l}{2}\right)$

= $\frac{1}{4}\pi$r (r + 4 l) Sq. units

4. Find the height of cone, if its slant height is 34 cm and base diameter is 32 cm

Sol. Let height of cone be h cm

Here, radius = $\frac{32}{2}$ = 16 cm

And slant height = 34 cm

∴ Height of the cone = $\sqrt{l^2 - r^2}$ = $\sqrt{(34)^2 - (16)^2}$

= $\sqrt{1156 - 256}$ = $\sqrt{900}$ = 30 cm

IV. Multiple choice questions

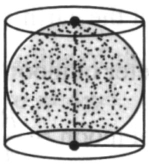
1. Then relation between surface area of a sphere and lateral surface area of a right circular cylinder that just encloses the sphere is

a) surface area of a sphere is greater than the lateral surface area of a right circular cylinder.

b) surface area of a sphere is less than the lateral surface area of a right circular cylinder.

c) surface area of a sphere is equal that of lateral surface area of a right circular cylinder.

Sol. c



2. A basket ball is just packed in a cube of side 20 cm, then the surface area of the basket ball is.

a) 120 cm^2 b) 2400 cm^2 c) 400 π cm^2 d) 1256 cm^2

Sol. c

3. The surface area of two hemispheres are in the ratio 25: 49. Find the ratio of their radii.

Sol. Let r and R be the radii of two hemispheres respectively

$\therefore \frac{Surface\ area\ of\ Ist\ hemisphere}{Surface\ area\ of\ 2nd\ hemisphere} = \frac{25}{49}$

$\Rightarrow \frac{3\pi r^2}{3\pi R^2} = \frac{25}{49}$

$\Rightarrow \frac{r^2}{R^2} = \frac{25}{49} \Rightarrow \frac{r}{R} = \frac{5}{7}$

∴ Ratio of the radii of two hemispheres = 5 : 7

4. If surface area of a sphere is 784 $\pi\ cm^2$, find its radius.

Sol. Let radius of sphere = r cm

Given surface area of sphere = 784 $\pi\ cm^2$

$\Rightarrow 4\pi r^2 = 784\ \pi$

$\Rightarrow 4r^2 = 784$

$\Rightarrow r^2 = \frac{784}{4} = 196 \Rightarrow$ r = 14 cm

∴ Radius of a sphere = 14 cm

5. Mehul does not like the colour on the wooden ball he has. So he wants to scratch and remove the colour so that he can put the new one. How much area he has to scratch, if diameter of ball is r cm?

Sol. Given diameter of ball = r cm

$\Rightarrow$ Radius of ball = $\frac{r}{2}$ cm

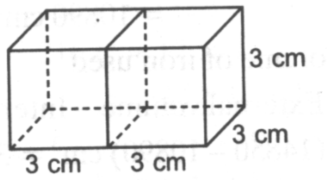
∴ Surface area of ball = $4\pi\left(\frac{r}{2}\right)^2 = \frac{4\pi r^2}{4} = \pi r^2\ cm^2$

V. Multiple choice questions

1. Two cubes of side 3 cm each are joined end to end, then the volume of the cuboid so formed is.

a) 32 cm^3 b) 16 cm^3 c) 54 cm^3 d) 8 cm^3

Sol. c



When two cubes of side 3 cm each are joined end to end then,

l = (3 + 3) cm = 6cm, b = 3 cm each are joined

∴ Volume (V) = $l$bh

V = 6 x 3 x 3 = 54 cm^3

2. Given a cuboid of dimensions l = 6cm, b = 5 cm h = 4 cm. How many cubes, each of side 2 cm can be but cut out from it?

a) 6 b) 15 c) 30 d) none of these

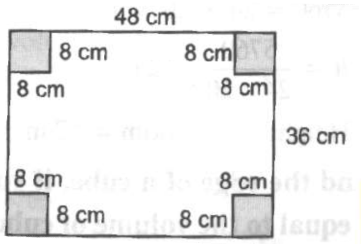
Sol: d

Breadth b = 5 is not divisible by 2, so we cannot cut out cubes.

3. A metallic sheet is of the rectangular shape with dimensions 48 cm x 36 cm. From each of its corner a square of 8 cm is cut off and an open box is made of the remaining sheet, then the volume of the box is

a) 5120 cm^2 b) 5012 cm^3 c) 5012 cm^2 d) 5120 cm^3

Sol : d



When squares of 8 cm is cut off, then

Length of the box = (48 – 16) cm = 32 cm

Breadth of the box =(36-16) cm = 20 cm

Height of the box = 8 cm

∴ Volume of the box = 32 x 20 x 8 = 5,120 cm^3

4. The volume of cube is 512 cm^3. Determine its edge.

Sol. Let edge of cube be x cm

Given volume of cube = 512 cm^3.

$\Rightarrow$ x^3 = 512 $\Rightarrow$ x = 8cm

5. From the given cuboid of dimensions l = 4cm, b = 2cm and h = 3 cm, how many cubes of edge 1 cm can be cut from it?

Sol. Given dimensions of cuboid are l = 4cm, b = 2cm and h = 3 cm

$\therefore$ Volume of cuboid = l x b x h = 4 x 2 x 3 cm^3

Given edge of cube = 1 cm

$\therefore$ Volume of cube = a^3 = $(1)^3$ = 1 cm^3.

$\therefore$ Number of cubes = $\frac{Volumes\ of\ cuboid}{Volume\ of\ cube} = \frac{4 \times 2 \times 3}{1}$ = 24

6. The length, breadth and height of a rectangular wooden box are 25 cm, 10.7 cm and 8.5 cm respectively. Find the volume of the box.

Sol. Given dimensions of a rectangular wooden box are

l = 25 cm, b = 10.7 cm, h = 8.5 cm

$\therefore$ Volume of the box = l x b x h

= 25 x 10.7 x 8.5

= 2273.75 cm^3

VI. Multiple choice questions

1. A conical tent is to accommodate 11 persons. Each person requires 4 square metres of the space on the ground and 20 cubic metres of air to breath, then the height of the cone is

a) 10 m b) 12m c) 15 m d) 18m

Sol. c

2. Ratio of the volume of a cone and a cylinder of same radius of base and same height is.

a) 1:1 b) 1:2 c) 1: 3 d) 1:4

Sol . c

3. A student has rectangular sheet of dimensions 14 cm x 22 cm. He wants to make a cylinder in such a way so that volume is minimum, then its height should be

a) 14 cm b) 22 cm c) 14cm or 22 cm d) none of these

Sol. b

4. In a Gym, in one exercise you have to continuously toss solid cylindrical jumbles and catch them. Cylindrical jumbles are of length 1 m and base diameter also 1m. Density of the wood used is 4 kg per m^3 then the weight tossed is.

a) 3.14 kg b) $\frac{\pi}{2}$ c) 4 π d) 12.56 kg

Sol. a

5. The radii of two cylinders are in the ratio 2:3 and their heights are in the ratio 2:3 and their heights are in the ratio 5:3. Find the ratio of their volumes.

Sol. Let radii of 1 st cylinder = $2x$ and radii of 2nd cylinder = $3x$

Let height of 1st cylinder= 5y and height of 2nd cylinder = 3y

$\therefore$ Ratio of their volumes = $\frac{\frac{1}{3}\pi\,(2x)^2 \times 5y}{\frac{1}{3}\pi(3x)^2 \times 3y}$ = $\frac{4x^2 \times 5y}{9x^2 \times 3y}$

= $\frac{20x^2y}{27x^2y}$ = $\frac{20}{27}$ = 20 : 27

6. If the radius of right circular cone is halved and its height is doubled, then the volume will remain unchanged. Is it true or false? Justify your answer.

Sol. Let radius of cone be r cm and height of cone be h cm

$\therefore$ Volume of cone = $\frac{1}{3}\,\pi r^2$h

Now, New radius = $\frac{r}{2}$ cm,

New height = 2h cm

$\therefore$ New volume of cone = $\frac{1}{3}\pi\,\left(\frac{r}{2}\right)^2$ x (2h)

= $\frac{1}{3}\pi$ x $\frac{r2}{4}$ x 2h = $\frac{1}{3}\left(\frac{1}{3}\,\pi r^2\mathrm{h}\right)$

Clearly the volume will change as the new volume is half of the original volume. Hence, the given statement is false.

VII. Multiple choice questions

1. If the radius (r) of a sphere is reduced to its half then, new volume would be:

a) $\frac{1}{2}\ (\frac{4}{3}\ \pi r^2)$ b) $\frac{4}{3}\ \pi r^2\ \frac{r^3}{2}$ c) $\frac{4}{3}\ \pi\ \frac{r^3}{8}$ d) $\frac{4}{6}\ \pi\ \frac{r^3}{8}$

Sol. C

2. If volume and surface area of a sphere is numerically equal then its radius is.

a) 2 units b) 3 units c) 4 units d) 5 units

Sol. b

3. A cylindrical jar is full of water upto the brim, a sphere of radius 0.35 cm is immersed in the water, how much water will flow out of the jar?

a) 0.18 cm^2 b) 0.18 cm^3 c) 1.54 cm^2 d) 1.54 cm^3

Sol. b

4. The volume of two hemispheres are in the ratio 27 : 125. Find the ratio of their radii

Sol. Let r_1 and r_2 be the radii of two hemispheres.

$$\frac{Volume\ of\ 1st\ sphere}{Volume\ of\ 2nd\ sphere} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3}$$

$$\frac{27}{125} = \left(\frac{r_1}{r_2}\right)^3$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{3}{5} \Rightarrow r_1 : r_2 = 3:5$$

5. If the radius of sphere is doubled. Find the ratio of volume of the new sphere to the original sphere.

Sol. Let radius of sphere be r

Volume of sphere = $\frac{4}{3}\ \pi r^3$

Radius of sphere is doubled

∴ Radius of new sphere = 2r

Volume of new sphere = $\frac{4}{3}\ \pi\ (2r)^3 = \frac{4}{3}\ \pi\ (8r^3)$

Ratio of volumes = $\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi\ (8r^3)} = \frac{1}{8}$

I Short Answer Questions (3 Marks)

1. Three equal cubes are placed adjacent to each other in a row. Find the ratio of the total surface area of the cuboid thus formed to the total surface area of the three cubes.

Sol. Let edge of three equal cubes be a

Surface area of one cube = $6a^2$

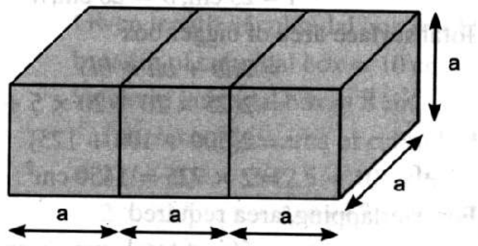
Total surface area of three cubes = 3 x $6a^2$ = $18a^2$

Three equal cubes are joined end to end, a cuboid is formed as shown.

Length of resulting cuboid = 3a

Breadth of resulting cuboid = a

Height of resulting cuboid = a



Surface area of resulting cuboid = 2 (lb + bh + hl)

= 2(3a x a + a x a + a x 3a)

= 2 ($3a^2$ + a^2 +$3a^2$) = $14a^2$

$\therefore \frac{Total\ surface\ area\ of\ resulting\ cuboid}{Total\ surface\ area\ of\ three\ cubes} = \frac{14a^2}{18a^2} = \frac{7}{9}$

Required ratio = 7: 9

10. A wall paper 312 m long and 25 cm wide is required to cover the walls of a room. Length of the room is 7 m and its breadth is twice its height. Determine the height of the room

Sol. Let height of room be h.

∴ Breadth = 2 x height

= 2h and length = 7 m

∴ Area of four wall = 2 (l + b)h

= 2 (7 + 2h)h ....(i)

∴ Area of four wall paper required to cover the four walls of the room.

= 312 x $\left(\frac{25}{100}\right)$ ....(ii)

From (i) and (ii), we have

$\frac{312 \times 25}{100}$ = 2 (7 + 2h) h

$\Rightarrow$ $\frac{312 \times 25}{100 \times 2}$ = (7 + 2h) h

$\Rightarrow$ 39 = (7 + 2h) h

$\Rightarrow$ $2h^2$ + 7h – 39 = 0

$\Rightarrow$ $2h^2$ + 7h - 39 = 0

$\Rightarrow$ $2h^2$ + 13h –6h - 39 = 0

$\Rightarrow$ $2h^2$ –6h + 13h - 39 = 0

$\Rightarrow$2h (h – 3) + 13 (h – 3) = 0

$\Rightarrow$ (h – 3) (2h + 3) = 0

$\Rightarrow$ h = 3 or h = $\frac{13}{2}$

(neglected as height is never negative)

$\Rightarrow$ h = 3 m

∴ Height of the room =3 m

II Short Answer Questions (2 Marks)

1. The curved surface area of a right circular cylinder is 4400 cm^2. If the circumference of the base is 110 cm, find the height of the cylinder.

Sol. Let height of cylinder be h cm

and radius of cylinder be r cm

Given circumference of the base of cylinder = 2 πr = 110 cm
and curved surface area of cylinder = 4400 cm^2

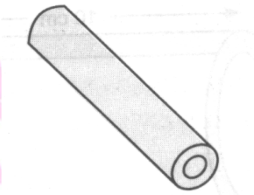
$\Rightarrow$ (2 πr)h = 4400 $\Rightarrow$ 110h = 4400 $\Rightarrow$ h = $\frac{4400}{110}$ = 40 cm

∴ Height of cylinder = 40 cm

2. **Savithri had to make a model of a cylindrical kaleidoscope for her science project. She wanted to use chart paper to make the curved surface of the kaleidoscope. (see figure). What would be the area of chart paper required by her, if she wanted to make a kaleidoscope of length 25cm with a 3.5 cm radius? You may take $\pi = \frac{22}{7}$**

Sol. Radius of the base of the cylindrical kaleidoscope (r) = 3.5 cm

Height (length) of kaleidoscope (h) = 25 cm



Area of chart paper required = curved surface area of the kaleidoscope

$$= 2\pi rh = 2 \times \frac{22}{7} \times 3.5 \times 25\ cm^2 = 550\ cm^2$$

3. **The inner diameter of a cylindrical vessel is 3.5 m. It is 100 m deep. Find the cost of polishing the inner curved surface area at the rate of ₹4 per m^2 (Use $\pi = \frac{22}{7}$)**

Sol. Given inner diameter of cylindrical vessel = 3.5 m

$\Rightarrow$ Inner radius of cylindrical vessel = $\frac{3.5}{2}$ m

$\Rightarrow$ Given depth of cylindrical vessel = 100 m

$\therefore$ Inner curved surface area of cylindrical vessel

$= 2\pi rh = 2 \times \frac{22}{7} \times \frac{3.5}{2} \times 100 = 1100\ m^2$

Given cost of polishing per m^2 = ₹4.

Total cost of polishing the curved surface area of 1100 m^2 = ₹4 x 1100 = ₹4400.

III Short Answer Questions (3 Marks)

1. **It costs ₹2200 to paint the inner curved surface of a cylindrical vessel 10m deep. If the cost of painting is at the rate of ₹20 per m^2, find the radius of the base.**

Sol. Given depth (h) of cylindrical vessel = 10m

Let radius of cylindrical vessel be r m

Curved surface area of cylindrical vessel = $2\pi rh = (2\pi r) \times 10\ m^2$

Given cost of painting per m^2 = ₹20

Cost of painting inner curved surface area = ₹2200

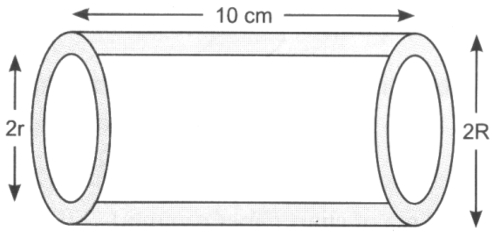
∴ 20 x (2 πr) x 10 = 2200

⟹ 20 x 2 x $\frac{22}{7}$ x r x 10 = 2200

⟹ r = $\frac{2200 \times 7}{20 \times 2 \times 22 \times 10}$ = 1.75 m

∴ Radius of base of a cylindrical vessel = 1.75m

2. The total surface area of hollow metal cylinder open at both ends of external radius 8 cm and height 10 cm is 338 π cm^2. Taking r to be inner radius, find the thickness of the metal in the cylinder.



Sol. Let r be the inner radius of a hollow metal cylinder.

Given external radius = R =8 cm and height = 10 cm

Total surface area of hollow metal cylinder

= (Outer + Inner curved surface area of cylinder) + Area of base rings

338 π = [2 πRh + 2 πrh] + 2π $(R^2 - r^2)$

338 π = 2 πh [R + r] + 2π $(R^2 - r^2)$

338 π = 2 x π x 10 x [8 + r] + 2π $(8^2 - r^2)$

338 π = π [20 [8 + r] + 2 $(64 - r^2)$]

338 = 160 + 20 r + 2 $(64 - r^2)$

⟹ $2r^2$ - 20r + 338 – 160 - 128 = 0

⟹ $2r^2$ - 20r + 50 = 0 ⟹ $2r^2$ - 10r - 10r + 50 = 0

⟹ 2r (r- 5) – 10 (r- 5) = 0 ⟹ (r- 5) (2r- 10) = 0

⟹ r = 5 cm

∴ Thickness of the metal in the cylinder = R – r = 8 – 5 = 3 cm

IV Short Answer Questions (2 Marks)

1. How many square metres of canvas is required for a conical tent whose height is 3.5 m and the radius of whose base is 12m?

Sol. Given radius (r) of the base of the cone = 12m

and height(h) of the cone is 3.5 m

∴ Slant height (l) of the cone

= $\sqrt{r^2+h^2}$ = $\sqrt{(12)^2+(3.5)^2}$

= $\sqrt{144+12.25}$ = $\sqrt{156.25}$ = 12.5 m

∴ Curved surface area of conical tent = $\pi r l$ sq.units.

= $\frac{22}{7}$ x 12 x 12.5 = 471.42 m^2

2. The diameters of two cones are equal. If their slant heights are in the ratio 7: 4, find the ratio of their curved surface area.

Sol. Let diameter of both comes be d.

Let radius = $\frac{d}{2}$ = r (say)

∴ Let slant height of first cone be $7x$ and slant height of second cone be $4x$

Let C_1 and C_2 be curved surface area of first and second respectively

∴ $\frac{C_1}{C_2}$ = $\frac{\pi r(7x)}{\pi r(4x)}$ = $\frac{7}{4}$ $\Longrightarrow$ C_1 : C_2 = 7 : 4

Therefore ratio of their curved surface area = 7:4.

V Short Answer Questions (3 Marks)

1. Anandita has a piece of canvas, whose area is 551 m^2. She used it to have a conical tent made with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting amount to 1 m^2. Find the slant height of conical tent.

Sol. Given radius of conical tent = r = 7 m

Let slant height of conical tent = l m

$\therefore$ Curved surface area of conical tent

= $\pi r l$ = $\frac{22}{7}$ x 7 x l = 22 l m^2

Given area of canvas sheet = 551 m^2

While cutting and stitching, area of canvas wastage is 1 m^2

$\therefore$ Area of canvas used for curved surface area of cone = 551 - 1 = 550 m^2

As per question,

550 = 22 $l \Rightarrow l = \frac{550}{22}$ = 25 m

$\therefore$ Slant height of cone = 25 m

2. A corn cob (see figure), shaped somewhat like a cone, has the radius of its broadest end as 2.1 cm and length (height) as 20 cm. If each 1 cm^2 of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob.

Sol. Since the grains of corn are found only on the curved surface of the corn cob, to we would need to know curved surface area of the corn cob to find the total number of grains on it. In this question, we are given the height of the cone, so we need to find its slant height.

Here, $l = \sqrt{r^2 + h^2}$ = $\sqrt{(2.1)^2 + 20^2}$ cm

= $\sqrt{404.41\ cm}$ = 20.11 cm

Therefore, the curved surface area of the corn cob = πrl sq. units

= $\frac{22}{7}$ x 2.1 x 20.11 cm = 132.726 cm^2

= 132.73 cm^2 (approx.)

VI Short Answer Questions (2 marks)

1. Charvi took a spherical orange and put a thread around its boundary in the middle. She noted that the length of the thread was 22 cm. How much do you think was the diameter of the orange?

Sol. Let radius of spherical orange = r cm

∴ Circumference from the middle of spherical orange

= Length of thread bound around the middle

$\Rightarrow 2\pi r = 22$

$\Rightarrow 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow \frac{2r}{7} = 1 \Rightarrow r = \frac{7}{2}$ cm

∴ Diameter of the orange = $2r = 2 \times \frac{7}{2}$ cm = 7 cm

3. A dome of the building is in the form of hemisphere. From inside, it was white washed at the cost of ₹ 498.96. If the cost of white washing is ₹ 2 per square metre, find the inner surface area of dome.

Sol. Given cost of white washing per m^2 = ₹2

And cost of white washing curved surface area of dome = ₹498.96

∴ Curved surface area = S = $\frac{498.96}{2}$ = 249.48 m^2

VII Short Answer Questions (3 marks)

1. If the total surface area of sphere is 98.56 cm^2, find the radius of the sphere.

Sol. Let radius of the sphere be r cm

Given surface area of sphere = 98.56 cm^2

$\Rightarrow 4\pi r^2 = 98.56\ cm^2$

$\Rightarrow r^2 = \frac{98.56 \times 7}{4 \times 22} = 7.84$

$\Rightarrow r^2 = 7.84 \Rightarrow 2.8$ cm ∴ Radius of the sphere = 2.8 cm

2. **The outer curved surface areas of hemisphere and sphere are in the ratio 2: 9. Find the ratio of their radii**

Sol. Let radius of the sphere be R cm and radius of hemisphere be r cm

$$\frac{Outer\ curved\ surface\ area\ of\ hemisphere}{Surface\ area\ of\ sphere} = \frac{2\pi r^2}{4\pi R^2}$$

$\frac{2}{9} = \frac{2r^2}{4R^2}$

$\Longrightarrow \frac{r^2}{R^2} = \frac{4}{9}$

$\Longrightarrow \frac{r}{R} = \frac{2}{3}$

∴ Required ratio of their radii = 2 : 3

VIII Short Answer Questions (2 marks)

1. **The volume of a committee room is 5760 m^3. Its length and breadth are 24 m and 20 m respectively. Find the height of the room.**

Sol. Volume of committee room = 5760 m^3

Length of committee room = l = 24m

Breadth of committee room = b = 20 m

Let height of committee room = h m

∴ Volume of committee room = l x b x h

$\Longrightarrow$ 5760 = 24 x 20 x h

$\Longrightarrow$ h = $\frac{5760}{24 \times 20}$ = 12 m

∴ Height of the room = 12 m

2. **Find the edge of a cube, if volume of the cube is equal to the volume of cuboid of dimensions. 8m x 4m x 2 m**

Sol. Let edge of cube be x m

∴ Volume of cube = x^3 m^3

Given dimensions of cuboid are l = 8m, b = 4m, h = 2m

∴ Volume of cuboid = l x b x h = 8 x 4 x 2 = 64 m^3

∴ According to question, volume of cube = Volume of cuboid

$\Longrightarrow$ x^3 = 64 $\Longrightarrow x$ = $(64)^{1/3}$ $\Longrightarrow x$ = 4 m

∴ Edge of the cube = 4 m

IX Short Answer Questions (3 marks)

1. A reservoir is in the form of a rectangular parallelopiped (cuboid). Its length is 20 m. If 18 kl of water is removed from the reservoir, the water is removed from the reservoir, the water level goes down by 15 cm. Find the width of the reservoir (1kl = 1 m^3)

Sol. Length of rectangular parallelepiped (cuboid) = 20 m

Volume of water removed from reservoir = 18 kl = 18 m^3 (1 kl = 1 m^3)

Level (height) of water in reservoir = 15 cm = $\frac{15}{100}$ m

Let width of reservoir = b m

∴ Volume of reservoir = l x b x h

$\Longrightarrow$ 20 x b x $\frac{15}{100}$ = 18 $\Longrightarrow$ b = $\frac{18 \times 100}{20 \times 15}$ = 6 m

Therefore width of the reservoir = 6 m.

2. How many cubic centimetres of iron are there in an open box whose external dimensions are 36 cm, 25 cm and 16.5 cm, the iron being 1.5 cm thick throughout? If 1 cubic cm of iron weighs 15 g, find the weight of the iron used in the box.

Sol. Given external dimensions of open box are

L = 36cm, B = 25 cm, H = 16.5cm

∴ External volume of open box

= L x B x H = 36 x 25 x 16.5 = 14850 cm^3

Given thickness of iron sheet throughout = 1.5 cm

∴ Internal length = l = 36 – (1.5 + 1.5)= 36 – 3 = 33cm

Internal breadth b = 25 – (1.5 + 1.5) = 25 - 3 = 22 cm

Internal height = h = 16.5 – 1.5 = 15 cm

Internal volume = l x b x h = 33 x 22 x 15 = 10890 cm^3

∴ Volume of iron used = External volume – Internal volume

= (14850 – 10890) cm^3 = 3960 cm^3

Given 1 cm^3 of iron weighs = 15 g

∴ 3960 cm^3 of iron weighs = 15 x 3960 = 59400 g = 59.4 kg

3. A box with lid is made of 2 cm thick wood. Its external length, breadth and height are 25 cm, 18 cm and 15 cm respectively. How many cubic cm of liquid can be placed in it? Also find the volume of the wood used in it.

Sol. Given external dimensions of box are

L = 25 cm, B = 18 cm, H = 15 cm

∴ External volume of box

= L x B x H = 25 x 18 x 15 = 6750 cm^3

Given thickness of wood = 2 cm

∴ Internal dimensions of box are l =25 – 4 = 21 cm,

b = 18 – 4 = 14 cm, h = 15 - 4 = 11 cm

∴ Volume of liquid that can be placed in box = Internal volume of box

= l x b x h = 21 x 14 x 11 = 3234 cm^3

∴ Volume of wood used = External volume – Internal volume

= 6750 – 3234 = 3516 cm^3

X Short Answer Questions (2 marks)

1. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the volume of cylinder.

Sol. Let radius of cylinder = r cm

Height of cylinder = 14 cm

Curved surface area of cylinder = 88 cm^2

$\Rightarrow$ $2\pi rh$ = 88

$\Rightarrow$ 2 x $\frac{22}{7}$ x r x 14 = 88

$\Rightarrow$ r = $\frac{88 \times 7}{2 \times 22 \times 14}$ = 1 cm

∴ Volume of cylinder = πr^2h = $\frac{22}{7}$ x 1 x 1 x 14 = 44 cm^3

2. **Determine the volume of a conical tin having radius of the base as 30 cm and its slant height as 50 cm (Use π = 3.14)**

Sol. Radius of base of conical tin = r = 30 cm

Slant height of conical tin = l = 50 cm

∴ Height of conical tin = h = $\sqrt{l^2 - r^2}$

$= \sqrt{(50)^2 - (30)^2}$

$= \sqrt{2500 - 900}$

$= \sqrt{1600}$ = 40 cm

∴ Volume of conical = $\frac{1}{3}\pi r^2$h cu. units

= $\frac{1}{3}$ x 3.14 x 30 x 30 x 40

= 314 x 3 x 40

= 37680 cm^3

3. **The volume of a cylindrical rod is 628 cm^3. If its height is 20 cm, find the radius of its cross section. (Use π = 3.14)**

Sol. Let radius of cross section of rod = r cm

Height of cylindrical rod = 20 cm

Volume of cylindrical rod = 628 cm^3

$\Rightarrow \pi r^2$h = 628

$\Rightarrow$ 3.14 x r^2 x 20 = 628

$\Rightarrow r^2 = \frac{628 \times 100}{314 \times 20}$ = 10

$\Rightarrow$ r = $\sqrt{10}$ cm = 3.16 cm

∴ Radius of its cross section = 3.16 cm

XI Short Answer Questions (3 marks)

1. **What is flowing at the rate of 3 km/hour through a circular pipe of 20 cm internal diameter into a circular pipe of 20 cm internal diameter into a circular cistern of diameter 10m and depth 2m. In how much time will the cistern be filled?**

Sol. Internal diameter of circular pipe = 20cm

$\Rightarrow$ Internal radius = r = $\frac{20}{2}$ = 10 cm = $\frac{10}{100}$ = $\frac{1}{10}$ m

Water is flowing at the rate of 3 km / hour = 3000m/hour

Volume of water flowing in one hour = πr^2h cu.units

$= \pi\left(\frac{1}{10}\right)^2$ x 3000 m^3

Diameter of cistern = 10 m

Radius of cistern = $\frac{10}{2}$ = 5 m

Depth of cistern = 2m

Volume of water in cistern = πr^2h = $\pi(5)^2$ x 2 m^3

Let the time taken to fill the cistern = t hours

$\therefore t = \frac{Volume\ of\ water\ in\ cistern}{Volume\ of\ water\ flowing\ from\ pipe\ in\ 1\ hour}$

$= \frac{\pi(5)^2 \times 2}{\pi\left(\frac{1}{100}\right) \times 3000}$ = $1\frac{2}{3}$ hours = 1 hour 40 minutes.

2. The barrel of a fountain - pen cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen is used up on writing 330 words on an average. How many words would use up a bottle of ink containing one fifth of a litre?

Sol. Length of cylindrical fountain pen = 7 cm = 70 mm

Diameter of cylindrical fountain pen = 5 mm

$\Rightarrow$ Radius of cylindrical fountain pen = $\frac{5}{2}$ mm

Volume of ink in cylindrical fountain pen = πr^2h cu. units

= $\pi \left(\frac{5}{2}\right)^2$ x 70 = $\frac{22}{7}$ x $\frac{25}{4}$ x 70 = 1375 mm^3

1375 mm^3 of ink is used to write 330 words

$\therefore \frac{1375}{10^6}$ l of ink is used to write 330 words ($1l$ = 10^6 mm^3)

$\Rightarrow \frac{1}{5} l$ of ink is used to write = $\frac{330 \times 1000000}{1375 \times 5}$ = 48000 words

3. Into a conical tent of radius 8.4 m and vertical height 3.5 m, how many full bags of wheat can be emptied, if space for the wheat in each bag is 1.96 m^3 ?

Sol. Radius of conical tent = 8.4 m

Height of conical tent = 3.5 m

$\therefore$ Capacity of the conical tent = $\frac{1}{3}$ πr^2h cu. units

= $\frac{1}{3}$ x $\frac{22}{7}$ x $(8.4)^2$ x 3.5 = 258.72 m^3

Space of occupied by two each bag of wheat = 1.96 m^3

$\therefore$ Number of bags

$= \frac{Capacity\ of\ the\ conical\ tent}{Space\ occupied\ by\ each\ bag\ of\ wheat} = \frac{258.72}{1.96}$ = 132.

XII Short Answer Questions (2 marks)

1. The diameter of a sphere is decreased by 25%. Find its new volume.

Sol. Let radius of sphere be r

Diameter of sphere = 2r

Volume of sphere = $\frac{4}{3} \pi r^3$

Diameter of sphere decreased by 25%

New diameter = (2r – 25% of 2r)

= (2r - $\frac{1}{4}$ x 2r) = $\frac{3}{4}$ x 2r = $\frac{3r}{2}$ $\left[25\% = \frac{25}{100} = \frac{1}{4}\right]$

New radius = $\frac{3r}{4}$

New volume of sphere = $\frac{4}{3} \pi \left(\frac{3r}{4}\right)^3$ = $\frac{4}{3}$ x $\frac{27}{64}$ πr^3 = $\frac{9}{16} \pi r^3$ cu. units

2. A solid sphere of radius 3 cm is melted and then recast into small spherical balls each of diameter 0.6 cm. Find the number of small balls thus obtained.

Sol. Radius of solid sphere = 3 cm

Volume of solid sphere = $\frac{4}{3}\pi(3)^3$

Diameter of small spherical ball = 0.6 cm

Radius of small spherical ball = $\frac{0.6}{2}$ = 0.3 cm

Volume of small spherical ball = $\frac{4}{3}\pi(0.3)^3\ cm^3$

Number of small spherical balls = $\frac{\frac{4}{3}\pi(3)^3}{\frac{4}{3}\pi(0.3)^3}$ = $\frac{3\times3\times3}{0.3\times0.3\times0.3}$ = 1000

3. The diameter of a sphere is 42 cm. It is melted and drawn into a cylindrical wire of 28 cm diameter. Find the length of wire.

Sol. Diameter sphere = 42 cm

Radius of sphere = $\frac{42}{2}$ cm = 21 cm

Diameter of cylindrical wire = 28 cm

Radius of cylindrical wire = $\frac{28}{2}$ = 14 cm

Let h be the length of cylindrical wire

Now , volume of sphere = volume of cylindrical wire

$\frac{4}{3}\pi(21)^3$ = $\pi\frac{4}{3}(14)^2$ x h

$\Rightarrow h$ = $\frac{4}{3}$ x = $\frac{21\times21\times21}{14\times14}$ = 63cm

4. A hemispherical bowl of internal diameter 36 cm contains a liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm. How many bottles are required to empty the bowl?

Sol. Internal radius of cylindrical bowl = $\frac{36}{2}$ = 18 cm

Radius of cylindrical bottle = 3cm

Height of cylindrical bottle = 6 cm

Let the number of required bottles be n

Volume of hemispherical bowl = n (volume of cylindrical bottles)

$\frac{2}{3}\pi(18)^3$ = n $[\pi(3)^2$ x 6]

$n = \frac{2\times 18\times 18\times 18}{3\times 9\times 6}$ = 72

XIII Short Answer Questions (3 marks)

1. Solid spheres of diameter 4 cm are dropped into a cylindrical beaker containing some water and are fully submerged. If the diameter of the beaker is 12 cm and the water rises by 24 cm, find the number of solid spheres dropped in the water.

Sol. Diameter of solid sphere = 4 cm

Radius of solid spheres = $\frac{4}{2}$ = 2 cm

Let number of spheres dropped in water be n

Volume of n spheres = n$\left[\frac{4}{3}\pi(2)^3\right]$

Diameter of cylindrical beaker = 12 cm

Radius of cylindrical beaker = $\frac{12}{2}$ = 6 cm

Height upto which the water level rises = 24 cm

Volume of water rises = $\pi\ (6)^2$ x 24 cm^3

As per question,

n $\left[\frac{4}{3}\pi(2)^3\right]$ = π x 36 x 24

n = $\frac{36\times 24\times 3}{4\times 8}$ = 81

2. A solid metallic sphere of diameter 21 cm is melted and recasted into a number of smaller cones, each of diameter 3.5cm and height 3 cm. Find the number of cones so formed.

Sol. Diameter of metallic sphere = 21 cm

Radius of metallic sphere = $\frac{21}{2}$ cm

Volume of metallic sphere = $\frac{4}{3}\pi\left(\frac{21}{2}\right)^2 cm^3$

Diameter of cone = 3.5 cm

Radius of cone = $\frac{3.5}{2}$ cm

Height of cone = 3 cm

Volume of cone = $\frac{1}{3}\pi\left(\frac{3.5}{2}\right)^2 \times 3cm^3$

Let number of cones be n

Volume of cone =n $\left[\frac{1}{3}\pi\left(\frac{3.5}{2}\right)^2 \times 3\right] cm^3$

As per question, volume of sphere = Volume of n cones

$\frac{4}{3}\pi\left(\frac{21}{2}\right)^3 = \frac{n}{3}\pi\left(\frac{3.5}{2}\right)^2 \times 3$

$\Rightarrow$ n = $\frac{4\times21\times21\times21\times3\times2\times2}{3\times2\times2\times2\times3\times3.5\times3.5}$ = 504

3. A hemispherical bowl of internal diameter 30 cm contains some liquids. This liquid is to be filled into cylindrical shaped bottles each of diameter 5 cm and height 6 cm Find the number of bottles necessary to empty the bowl.

Sol. Internal diameter of hemispherical bowl = 30 cm

Internal radius of hemispherical bowl = R = $\frac{30}{2}$ = 15 cm

∴ Volume of liquid in hemispherical bowl

= $\frac{2}{3}\pi R^3$ = $\frac{2}{3}\pi \times (15)^3 cm^3$

Diameter of cylindrical bottle = 5 cm

Radius of cylindrical bottle = r = $\frac{5}{2}$ cm

Height of cylindrical bottle = h = 6 cm

Let number of cylindrical shaped bottles = n[πr^2h]

= n $\left[\pi\left(\frac{5}{2}\right)^2 \times 6\right]$

As per question volume of liquid in hemispherical bowl and n cylindrical bottle remains same.

∴ n $\left[\pi\frac{25}{4} \times 6\right] = \frac{2}{3}\pi \times (15)^3$

n = $\frac{2\times15\times15\times15\times4}{3\times25\times6}$ = 60

∴ Required number of bottles to empty the bowl = 60

I Long Answer Questions

1. A class room is 7 m long, 6.5 m wide and 4 m high. It has one door 3 m x 1.4 m and three windows each measuring 2 m x 1 m. The interior walls are to be colour washed, the contractor charges ₹5.25 per m^2. Find the cost of colour washing.

Sol. Given dimensions of classroom are

L = 7m, b = 6.5 m, h = 4 m

∴ Area of four walls of the room

= 2 (l + b) h

= 2 (7 + 6.5) x 4

= 2 x 13.5 x 4 = 108 m^2

Area of one door = 3 m x 1.4 m = 4.2 m^2

∴ Area of three windows = 3 x (2 m x 1 m)

= 3 x ($2m^2$) = 6 m^2

Remaining area of walls to be colour washed = (Area of four walls)

∴ [Area of one door + Area of three windows]

= 108 – (4.2 + 6)
= 108 – 10.2 = 97.8 m^2

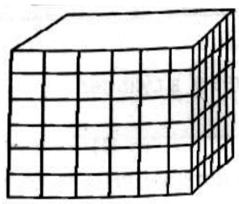
Cost of colour washing per m^2 = ₹5.25

Cost of colour washing 97.8 m^2 = ₹5.25 x 97.8

= ₹ 513.45

2. Hameed has built a cubical water tank with lid for his house, with each outer edge 1.5 m long. He gets the outer surface of the tank excluding the base, covered with square tiles of side 25 cm. Find how much he would spend for the tiles, if the cost of the tiles is ₹ 360 per dozen.

Sol. Since Hameed is getting the five out faces of the tank covered with tiles, he would need to know the surface area of the tank, to decide on the number of tiles required.
Edge of the cubical tank = 1.5 m = 150 cm (= a)
So surface area of the tank = 5 x 150 x 150 cm^2
Area of each square tile = side x side = 25 x 25 cm^2



So, the number of tiles required

$= \frac{Surface\ area\ of\ the\ tank}{area\ \ of\ each\ tile} = \frac{(5 \times 150 \times 150)}{25 \times 25} = 180$

Cost of 1 dozen tiles, i.e. cost of 12 tiles = ₹360

Therefore , cost of one tile = $\frac{360}{12}$ = ₹30

So the cost of 180 tiles = 180 x ₹ 30 = ₹ 5400

II Long Answer Questions

1. Circumstances of the base of a cylinder open at the top is 132 cm. The sum of radius and height of the cylinder is 41 cm. Find the cost of polishing the outer surface of cylinder at the rate of ₹10 per square metre.

Sol. Let radius of cylinder be r cm

Circumference of the base of the cylinder = $2\pi r$

$\Rightarrow\ 2\pi r = 132$

$\Rightarrow\ 2 \times \frac{22}{7} \times r = 132 \Rightarrow r = \frac{132 \times 7}{2 \times 22} = 21cm$

Let height of the cylinder be h cm

Also r + h = 41 cm $\Rightarrow$ 21 + h = 41 $\Rightarrow$ h = 20 cm

∴ Outer curved surface area of cylinder

$= 2\pi rh = 2 \times \frac{22}{7} \times 21 \times 20 = 2640\ cm^2$

∴ Outer base area = $\pi r^2 = \frac{22}{7} \times 21 \times 21 = 1386\ cm^2$

Total outer surface area = (2640 + 1386) = 4026 cm^2

Given cost of polishing per m^2 = ₹10

∴ Total cost of polishing = $\frac{₹10 \times 4026}{10000}$ = ₹4026

10. A cylindrical roller 2.5 m in length 1.5 m in radius when rolled on road was found to cover the area of 16500 m^2. How many revolutions does it make?

Sol. Given radius of cylindrical roller = 1.5 m and height of cylindrical roller = 2.5m

∴ Area covered in one revolution

= Curved surface area of cylinder

= $2\pi rh = 2 \times \frac{22}{7} \times 1.5 \times 2.5\ m^2$

Let in 'n' number of revolutions, area covered is $16500 m^2$

Hence, $n \times \left(2 \times \frac{22}{7} \times 1.5 \times 2.5\right) = 16500$

$N = \frac{16500 \times 7}{2 \times 22 \times 1.5 \times 2.5} = 700$

∴ A cylindrical roller makes 700 revolutions.

III Long Answer Questions

1. There cubes of metal whose edges are in the ratio 3:4:5 are melted down into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes.

Sol. Given ratio of the edges of three cubes = 3: 4 : 5

Let Edge of 1st cube = $3x$ cm

Edge of 2nd cube = $4x$ cm

Edge of 3rd cube = $5x$ cm

∴ Total volume of all the three cubes

$= (3x)^3 + (4x)^3 + (5x)^3$

$= 27x^3 + 64x^3 + 125x^3$

$= 216x^3\ cm^3$

Let edge of new cube formed by y cm

∴ Length of diagonal of new cube = $\sqrt{3}$ y

$\Rightarrow \sqrt{3}y = 12\sqrt{3}$

(∵ Given diagonal of new cube = $12\sqrt{3}$ cm)

$\Rightarrow$ Y = 12 cm

$\therefore$ Volume of new cube = $(12)^3\ cm^3$

According to question

$216\,x^3 = (12)^3$

$\Rightarrow 216\,x^3 = 12 \times 12 \times 12$

$\Rightarrow x^3 = \frac{12 \times 12 \times 12}{216} = 2 \times 2 \times 2$

$\Rightarrow x^3 = 2 \times 2 \times 2$

$\Rightarrow x = 2$ cm

$\therefore$ Edge of 1st cube = $3x$ = 3 x 2 = 6 cm

Edge of 2nd cube = $4x$ = 4 x 2 = 8 cm

Edge of 3rd cube = $5x$ = 5 x 2 = 10 cm

13. A wall was to be built across an open ground to cover a width (or breadth) of 10 m. The height of the wall is 4 m and thickness of the wall is 24 cm. If this wall is to be built up with bricks whose dimensions are 24 cm x 12 cm x 8 cm, how many bricks would be required?

Sol. Given dimensions of the wall are

Width (breadth) of wall = 10 m

Height of wall = 4 m

Thickness = 24 cm = $\frac{24}{100}$ m

Total space occupied by wall

= Thickness x Width x Height = $\frac{24 \times 10 \times 4}{100}\ m^3$

Given dimensions of brick are l = 24 cm = $\frac{24}{100}$ m

b = 12 cm = $\frac{12}{100}$ m , h = 8 cm = $\frac{8}{100}\ m^3$

$\therefore$ Space occupied by one brick = l x b x h = $\frac{24}{100} \times \frac{12}{100}$

$\therefore$ Number of bricks required

$= \frac{Space\ occupied\ by\ wall}{Space\ occupied\ by\ one\ brick} = \frac{24 \times 10 \times 4}{100} \times \frac{100 \times 100 \times 100}{24 \times 12 \times 8} = 4166.66 \approx 4167$ bricks

14. The external length, breadth and height of a closed rectangular wooden box are 18 cm, 10 cm and 6 cm respectively and thickness of wood is 0.5 cm, When the box is empty, it weighs 15 kg and when filled with sand, it weighs 100 kg. Find the weight of 1 cm^3 of the wood and sand.

Sol. Given dimensions of closed wooden box are

L = 18 cm B = 10cm H = 6 cm

∴ External volume of box

= L X B X H = 18 x 10 x 6 = 1080 cm^3

Given thickness of wood = 0.5 cm

∴ Internal dimensions of box are

l = 18 – (0.5 + 0.5) = 18 -1 = 17 cm, b = 10 – (0.5 + 0.5) = 10 - 1 = 9 cm

h =6 – (0.5 + 0.5) = 6 -1 = 5 cm

∴ Internal volume of box = l x b x h = 17 x 9 x 5 = 765 cm^3

∴ Volume of wood used

= External volume – Internal volume

= 1080 - 765 = 315 cm^3

∴ 315 cm^3 of wood weighs = 15 kg

1 cm^3 of wood weighs = $\frac{15}{315} = \frac{1}{21}$ kg

When box is filled with sand, its total weight is 100kg

∴ Weight of sand only

= Total weight of box when filled with sand - Weight of empty box

= 100 – 15 = 85 kg

∴ 765 cm^3 of sand weighs = 85 kg

1 cm^3 of sand weighs = $\frac{85}{765} = \frac{1}{9}$ kg.

15. A teak wood log is cut first in the form of cuboid of length 2.3 m, width 75 cm and of certain thickness. Find its thickness, if its volume is 1.104m^3. How many rectangular planks of size 2.3 m x 75 cm x 4 cm can be cut from the cuboid?

Sol. Given dimensions of teak wood log (cuboidal shape) are l = 2.3 m, b = 75 cm = 0.75 m

Let thickness = t m

Given volume of teak wood log = 1.104 m^3

$\Rightarrow l \times b \times h = 1.104 \Rightarrow 2.3 \times 0.75 \times t = 1.104$

$\Rightarrow t = \frac{1.104}{2.3 \times 0.75}$ = 0.64 m

Given dimensions of rectangular planks are l = 2.3 m

B = 75 cm = 0.75 m, h = 4 cm = 0.04 m

∴ Number of rectangular planks

$= \frac{Volume\ of\ teak\ wood\ log}{Volume\ of\ rectangular\ planks}$

$= \frac{1.104}{2.3 \times 0.75 \times 0.04} = 16$

16. A wall 6 m long, 5 m high and 0.5 m thick is to be constructed with bricks, each having length 25 cm, breadth 12 cm and height 7.5 cm. Find the number of bricks required to construct the wall. If it is given that cement and sand mixture occupy $\frac{1}{20}$ of the volume of the wall.

Sol. Given dimensions of wall are

L = 6m, b = 0.5 m, h = 5m

Volume of wall = l x b x h = 6 x 0.5 x = 15 m^3

Volume of cement and sand mixture

$= \frac{1}{20}$ of 15 $= \frac{15}{20} = \frac{3}{4} m^3$

∴ Volume of bricks used

$= 15 - \frac{3}{4} = \frac{57}{4} m^3 = 14.25\ m^3$

Given dimensions of brick are l = 25 cm = 0.25 m,

B = 12.5 cm = 0.125 m, h = 7.5 cm = 0.075 m

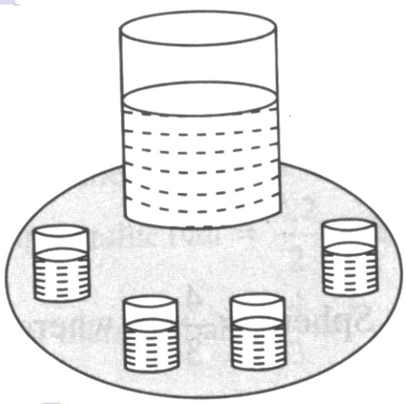
Volume of one brick = l x b x h = 0.25 x 0.125 x 0.075 m^3

Number of bricks = $\frac{Volume\ of\ bricks\ used}{Volume\ of\ one\ brick}$

= $\frac{14.25}{0.25\ \times 0.125 \times 0.075}$ = 6080

IV Long Answer Questions

1. **At a Ramzan Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small cylindrical glasses (see figure) of radius 3 cm up to a height of 8 cm, and sold for Rs.3 each. How much money does the stall keeper receive by selling the juice completely?**



Sol. Radius of large cylindrical vessel = R = 15 cm

Height of large cylindrical vessel = H = 32 cm

∴ Volume of juice in large cylindrical vessel = πR^2H

= $\pi\ (15^2)$ x 32 cm^3

Radius of small cylindrical glass = r = 3 cm

Height of small cylindrical vessel = H = 8 cm

∴ Volume of juice in each glass = πR^2H = $\pi\ (3^2)$ x 8 cm^3

Number of glasses of juice that are sold

= $\frac{\pi\ (15^2)\ \text{x}\ 32}{\pi\ (3^2)\ \text{x}\ 8}$ = 5 x 5 x 4 = 100

Selling price of one glass = ₹ 3

∴ Total, money received by selling 100 glass.

₹ 3 x 100 = ₹ 300

2. **A cylindrical road roller made of iron is 1m long. Its inner diameter is 54 cm and the thickness of the iron sheet rolled into the road roller is 9 cm Find the weight of the roller, if 1 cm^3 of iron weighs 8 g (Use π = 3.14)**

Sol. Length of cylindrical road roller r = 5 = 1 m = 100 cm

Inner radius = r = $\frac{54}{3}$ = 27 cm

Thickness of iron sheet = 9cm

∴ Outer Radius R = 27 + 9 = 36 cm

∴ Volume of iron sheet = $\pi(R^2 - r^2)$ x h

= 3.14 x [$(36)^2$- $(27)^2$] x 100

= 3.14 x [$(1296 - 729)$ x 100 = 3.14 x 567 x 100

= 178038 cm^3

1 cm^3 of iron weighs = 8g

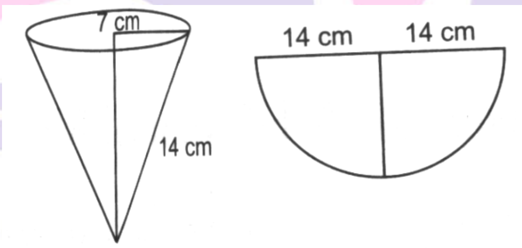
∴ 178038 cm^3 of iron weighs

= 8 x 178038 = 1424304 g = 1424. 304 kg

∴ Weight of roller = 1424.304 kg

3. A semi – circular sheet of metal of diameter 28 cm is bent to form an open conical cup. Find the capacity of the cup.

Sol. According to question



Circumferences of conical cup 2 πr= 14π ⟹ r = $\frac{14\pi}{2\,\pi}$ ⟹ r = 7 cm

∴ Height of conical cup

H = $\sqrt{l^2 - r^2}$ = $\sqrt{14^2 - 7^2}$

= $\sqrt{196 - 49}$ = $\sqrt{147}$ = 12.12 cm

∴ Volume of the conical cup = $\frac{1}{3}\pi r^2$h

= $\frac{1}{3}$ x $\frac{22}{7}$ x 7 x 7 x 12.12 = 622.16 cm^3

V Long Answer Questions

1. The volumes of two spheres are in the ratio 64 : 27. Find their radii, if the sum of their radii is 21cm.

Sol : Let r_1, cm be the radius of 1st sphere and r_2 cm be the radius of 2nd sphere

Also $r_1 + r_2$ = 21 cm

$\frac{Volume\ of\ 1\ st\ sphere}{Volume\ of\ end\ so\ here} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3}$

$\Rightarrow \frac{64}{27} = \frac{r_1^3}{r_2^3}$

$\Rightarrow \frac{r_1}{r_2} = \frac{4}{3} \Rightarrow \frac{4}{3}r_2$

Putting $r_1 = \frac{4}{3}r_2$ in (i) , we get

$\frac{4}{3}r_2 + r_2 = 21$

$\Rightarrow \frac{7r_2}{3} = 21 \Rightarrow r_2 = \frac{21 \times 3}{7} = 9$cm

$\Rightarrow r_1 = 21 - 9 = 12$ cm

∴ Radius of sphere = 12 cm

Radius of 2nd sphere = 9 cm

2. The largest sphere is carved out of a cube of side 7 cm. Find the volume of the sphere.

Sol. Side of cube = 7 cm

Diameter of sphere = Side of cube = 7 cm

Radius of sphere = r = $\frac{7}{2}$ cm

∴ Volume of sphere = $\frac{4}{3}\pi r^3 = = \frac{4}{3}\pi\left(\frac{7}{2}\right)^3$

$\frac{4}{3} \times \frac{22}{7} \times \frac{7 \times 7 \times 7}{2 \times 2 \times 2} = 179.67\ cm^3$

3. The total cost of making a spherical ball is ₹ 33,957 at the rate of ₹ 7 per cubic metre what will be the radius of this ball?

Sol. Let radius of the ball be r m

∴ Volume of spherical ball = $\frac{4}{3}\pi r^3$

Total cost of making a spherical ball at the rate of ₹ 7 per m^3 = ₹ 33957

∴ Volume of spherical ball = $\frac{33957}{7} = 4851m^3$

Now, $\frac{4}{3}\pi r^3$ = 4851

$\Rightarrow \quad r^3 = \frac{4851 \times 7 \times 3}{4 \times 22}$

$\frac{441 \times 21}{8} = \left(\frac{21}{2}\right)^3$

$\Rightarrow r^3 = \left(\frac{21}{2}\right)^3 \Rightarrow$ r $= \frac{21}{2}$ = 10.5 m

∴ Radius of spherical ball = 10.5 m